Applications of Damage Poromechanics to Geostorage and Injection Problems

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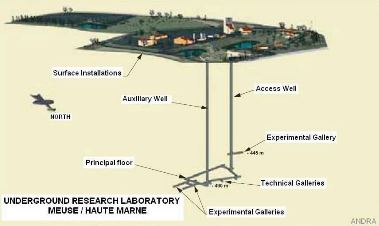
Research Seminar at ConocoPhillips, Houston, TX

Research Goals

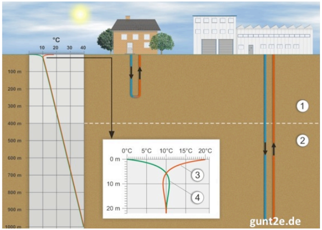
Effects of pore and crack interactions on damage and healing in rock

⇒ nuclear waste disposals, high-pressure gas storage



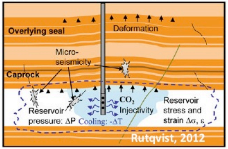
Thermo-hydro-mechanical couplings in soils due to cyclic thermal loadings

⇒ heat-exchanger piles, deep geothermal systems



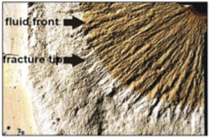
Surface and interface energy in porous media subjected to coupled THMC processes

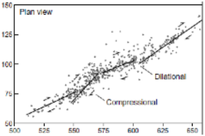
⇒ carbon dioxide sequestration, permafrost evolution due to climate change



Comparison of continuum and particulate mechanics for the assessment of damage

⇒ fluid injection, hydraulic fracturing





The Research Engine..

Expertise acquired...

Collaborations needed...

Theoretical Framework

- Thermodynamics of Porous Media
- Continuum Damage Mechanics

- thermo-hydro-chemo-mechanical processes in porous media
- effects of crack opening, closure and healing
- relationship between rock microstructure and macroscopic poro-elastic and flow properties

Numerical Tools

- Resolution algorithms at the Gauss Point (MATLAB)
- Finite Element code development ("Theta-Stock", in Fortran)
- Subroutine implementation in commercial software (ANSYS, Abaqus)

Experimental procedures

- Experimental plan to validate or reject modelling assumptions
- Test design (apparatus, stress and temperature ranges, sample preparation…)
- Laboratory expertise

Observation Scale(s)...

Laboratory Sample Scale : $0.1m$
“Representative Elementary Volume” (REV)

Microstructure “Observations” : $0.01\mu m$ - $10\mu m$
“Pore Size Distribution” (PSD)

Observation Scale(s)...

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(REV)

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$\Rightarrow$

Macroscopic thermodynamic variables
$\epsilon_{ij}, \quad p_l, \quad p_g, \quad T, \quad \Omega_{ij}...$

Microscopic geometric parameters
$r_{min}, \quad r_{max}, \quad r_{av}, \quad \lambda...$

$\Leftarrow$

Applications of Damage Poromechanics to Energy Geotechnics

1. **Modeling Damage in Porous Media By A Phenomenological Approach [Arson & Gatmiri, 2008-2011]**
2. **Determining Internal Variables to Model Rock Damage and Healing [Xu, Arson & Chester, 2012 ; Arson, Xu & Chester, 2012]**
3. **Relating Damaged Rock Microstructure to Macroscopic Flow and Poro-Elastic Properties [Arson & Pereira ; Pereira & Arson, 2012]**
4. **Homogenized Stiffness Tensor of Damaged Dentin Repaired by Resin Injection [Vennat & Arson, 2012]**

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   - Results : Simulation of Triaxial Compression Tests
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Thermodynamics of Open Systems [Coussy, 2004]

- First Law of Thermodynamics (porous solid filled with a non reactive fluid mixture)

$$\dot{\mathbb{K}} + \dot{\mathbb{E}}_{int} = \dot{\mathbb{E}}_{tot} = \mathbb{P}_{mec} + \dot{\mathbb{E}}_{chem} + \dot{\mathbb{Q}}$$

$$\mathbb{P}_{mec} = \mathbb{P}_{defo} + \dot{\mathbb{K}} \Rightarrow \dot{\mathbb{E}}_{int} = \mathbb{P}_{defo} + \sum_j \mu_j \dot{N}_j + \dot{\mathbb{Q}}$$

$$\Psi = \mathbb{E}_{int} - TS \Rightarrow \dot{\Psi} + T\dot{S} + S\dot{T} = \mathbb{P}_{defo} + \sum_j \mu_j \dot{N}_j + \dot{\mathbb{Q}}$$

- Second Law of Thermodynamics

$$\dot{S} \geqslant \frac{\dot{\mathbb{Q}}}{T}$$

- Inequality of Clausius-Duhem

$$\Phi = \mathbb{P}_{defo} + \sum_j \mu_j \dot{N}_j - S\dot{T} - \dot{\Psi} \geqslant 0$$

Thermodynamic potentials depend on state variables and internal variables.

State Variables for Non-Isothermal Unsaturated Porous Media

2 miscible pure fluids (liquid, gas), small deformation, absence of body forces.

Inequality of Clausius-Duhem :

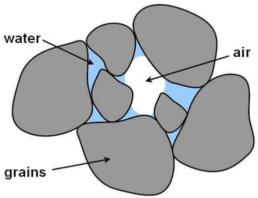
$$\Phi = \Phi_s + \Phi_l + \Phi_g + \Phi_T \geqslant 0$$

$$\Phi_s = \boldsymbol{\sigma} : \frac{d\epsilon}{dt} + p_l \frac{d^s \phi_l}{dt} + p_g \frac{d^s \phi_g}{dt} - S_s \frac{dT}{dt} - \frac{d^s \Psi_s}{dt}$$

$$\Phi_l = [-\boldsymbol{\nabla}_X p_l] \cdot \phi_l \mathbf{V_l^r}$$

$$\Phi_g = [-\boldsymbol{\nabla}_X p_g] \cdot \phi_g \mathbf{V_g^r}$$

$$\Phi_T = -\frac{\mathbf{Q}}{T} \cdot \nabla_X T$$



"deformation"	"stress"
$\boldsymbol{\epsilon^e}$	$\boldsymbol{\sigma}$
ϕ_l^e	p_l
ϕ_g^e	p_g
T	S_s

Thermodynamic Conjugation Relationships

The free energy of the solid skeleton is sought in the form :

$$\Psi_s = \Psi_s(\epsilon^e, \phi_l^e, \phi_g^e, T; \boldsymbol{\chi})$$

$\boldsymbol{\chi}$: vector containing all internal variables of interest, e.g. damage ($\boldsymbol{\Omega}$), hardening variables such as the equivalent plastic strain (γ^p)

Mechanical dissipation inequality (in the absence of plastic porosity changes) :

$$\boldsymbol{\sigma} : \frac{d\epsilon^p}{dt} + \left(\boldsymbol{\sigma} - \frac{\partial \Psi_s}{\partial \boldsymbol{\epsilon}^e}\right)\frac{d\epsilon^e}{dt} + \left(p_l - \frac{\partial \Psi_s}{\partial \phi_l^e}\right)\frac{d\phi_l^e}{dt} + \left(p_g - \frac{\partial \Psi_s}{\partial \phi_g^e}\right)\frac{d\phi_g^e}{dt} - \left(S_s + \frac{\partial \Psi_s}{\partial T}\right)\frac{dT}{dt} - \frac{\partial \Psi_s}{\partial \boldsymbol{\chi}}\frac{d\boldsymbol{\chi}}{dt} \geqslant 0$$

In the absence of irreversible microstructure change...

$$\left(\boldsymbol{\sigma} - \frac{\partial \Psi_s}{\partial \boldsymbol{\epsilon}^e}\right)\frac{d\epsilon^e}{dt} + \left(p_l - \frac{\partial \Psi_s}{\partial \phi_l^e}\right)\frac{d\phi_l^e}{dt} + \left(p_g - \frac{\partial \Psi_s}{\partial \phi_g^e}\right)\frac{d\phi_g^e}{dt} - \left(S_s + \frac{\partial \Psi_s}{\partial T}\right)\frac{dT}{dt} = 0$$

... from which thermodynamic conjugation relationships are deduced...

$$\boldsymbol{\sigma} = \frac{\partial \Psi_s}{\partial \boldsymbol{\epsilon}^e}; \quad p_l = \frac{\partial \Psi_s}{\partial \phi_l^e}; \quad p_g = \frac{\partial \Psi_s}{\partial \phi_g^e}; \quad S_s = -\frac{\partial \Psi_s}{\partial T}$$

... and the reduced dissipation inequality is obtained :

$$\boldsymbol{\sigma} : \frac{d\epsilon^p}{dt} + \boldsymbol{\xi} \cdot \frac{d\boldsymbol{\chi}}{dt} \geqslant 0 \quad ; \quad \boldsymbol{\xi} = -\frac{\partial \Psi_s}{\partial \boldsymbol{\chi}}$$

Choice of the stress state variables for unsaturated porous media

- **number of stress variables ?**
  A long-standing debate [Fredlund and Morgenstein 1977, Houlsby 1997]
  Example for incompressible solid grains :
  - Bishop's effective stress : $\boldsymbol{\sigma}' = (\boldsymbol{\sigma} - p_g\boldsymbol{\delta}) + \chi\,(p_g - p_l)\,\boldsymbol{\delta}$
  - independent state variables :
    $(\boldsymbol{\sigma} - p_g\boldsymbol{\delta};\ p_g - p_l)$ or $(p_g - p_l;\ \boldsymbol{\sigma} - p_l\boldsymbol{\delta})$ or $(\boldsymbol{\sigma} - p_l\boldsymbol{\delta};\ \boldsymbol{\sigma} - p_g\boldsymbol{\delta})$

- **nature of the state variables ?**
  No unique formulation [Coussy, 2004 ; Dangla, 2010].
  Example for elastic isothermal unsaturated porous media,
  based on the separation of energies :

$$\Psi_s\left(\boldsymbol{\epsilon}^e, \phi, S_l\right) = \psi_s\left(\boldsymbol{\epsilon}^e, \phi\right) + \phi\, U\,(\phi, S_l)$$

$$\begin{cases} \boldsymbol{\sigma} = \frac{\partial \psi_s\left(\boldsymbol{\epsilon}^e, \phi\right)}{\partial \boldsymbol{\epsilon}^e} \\ \pi = S_l p_l + S_g p_g - \frac{\partial(\phi\, U(\phi, S_l))}{\partial \phi} = \frac{\partial \psi_s\left(\boldsymbol{\epsilon}^e, \phi\right)}{\partial \phi} \\ p_c = -\frac{\partial U\left(\boldsymbol{\epsilon}^e, \phi_l, \phi_g\right)}{\partial S_l} \end{cases}$$

Damage Variable

$$\Omega_{ij} = \frac{1}{V_{REV}} \sum_{K=1}^{N} \left(l_K{}^3 \, n^K{}_i \, n^K{}_j \right)$$

- **Assumption 1 :** cracks do **not interact**
- **Assumption 2 :** the loss of deformation energy due to shear cracking is negligible, i.e. **cracks mainly open due to tensile (micro-)stress** (opening orthogonal to the crack plane)
- **Assumption 3 :** at the scale of the REV, damage may be represented by three **equivalent cracks** (homogenization)

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$$\Delta W_e = \frac{8(1-\nu_0^2)}{3(1-\nu_0/2)E_0} \left[(\sigma_{ij}\sigma_{jl}) \, \Omega_{li} - \frac{\nu_0}{2} \sigma_{ji} \, \frac{1}{V_{REV}} \sum_K \left(l_K{}^3 \, n^K{}_i n^K{}_j n^K{}_k n^K{}_l \right) \sigma_{lk} \right]$$

- **Assumption 2 :** the loss of deformation energy due to shear cracking is negligible, i.e. **cracks mainly open due to tensile (micro-)stress** (opening orthogonal to the crack plane)

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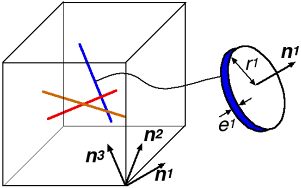
$$\Omega_{ij} = \sum_{k=1}^{3} \left(d_k \, n^k{}_i \, n^k{}_j \right)$$



THHMD Model [Arson, 2009] : Independent State Variables

1. **Assumption :** incompressible solid phase. Clausius-Duhem Inequality :

$$\left(\sigma_{ij} - p_a\delta_{ij}\right)\,\Delta\epsilon_{ji} + \left(p_a - p_w\right)\Delta\left(-nS_w\right) - \eta\Delta T \;-\Delta\Psi_s\left(\epsilon_{ij}, nS_w, T, \Omega_{ij}\right) \geq 0$$

2. **3 independent strain variables :** mechanical strain $\epsilon_{M_{ij}}$, capillary strain ϵ_{Sv} and thermal strain ϵ_{Tv}...
...conjugate to 3 independent stress variables :
net stress $\sigma"_{ij} = \sigma_{ij} - p_a\delta_{ij}$, suction $s = p_w - p_a$, and thermal stress p_T :

$$\begin{cases} \sigma"_{ij} \leftrightarrow \epsilon_{M\,ij} \\ s \leftrightarrow \epsilon_{Sv} \\ p_T \leftrightarrow \epsilon_{Tv} \end{cases}$$

3. **Thermodynamic decomposition of the total strain tensor :**

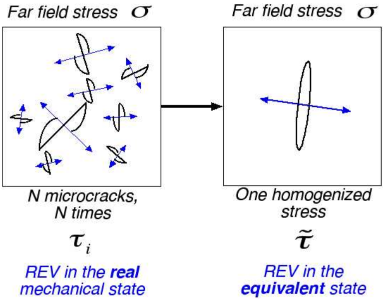
$$d\epsilon_{ij} = d\epsilon^e_{M_{ij}} + d\epsilon^d_{M_{ij}} + \frac{1}{3}\delta_{ij}\left(d\epsilon^e_{Sv} + d\epsilon^d_{Sv}\right) + \frac{1}{3}\delta_{ij}\left(d\epsilon^e_{Tv} + d\epsilon^d_{Tv}\right)$$

e : elastic, d : non-elastic (irreversible)

4. Clausius-Duhem Inequality written in terms of **stress/strain products :**

$$\sigma"_{ij}\,\Delta\epsilon_{M_{ij}} + s\,\Delta\epsilon_{Sv} + p_T\,\Delta\epsilon_{Tv} \;-\Delta\Psi_s\left(\epsilon_{M_{ij}}, \epsilon_{Sv}, \epsilon_{Tv}, \Omega_{ij}\right) \geq 0$$

Equivalent Mechanical State



- micro-stresses (opening the cracks in tension) :

$$\tau_{ij} = \sum_{k=1}^{3} \left(\tau^k \, {n^k}_i \, {n^k}_j \right)$$

- **in the equivalent mechanical state** [Swoboda & Yang, 1999] :

$$\widetilde{\tau}_{ij} = g \, \Omega_{ij}$$

Equivalent Mechanical State & Free Energy - Mechanical Problem

- Conjugation relationships with equivalent stress [Arson & Gatmiri, 2010] :

$$\widetilde{\sigma}_{ij} = \sigma_{ij} + \widetilde{\tau}_{ij} = \sigma_{ij} + g\,\Omega_{ij}, \qquad \widetilde{\sigma}_{ij} = \frac{\partial \Psi_e(\epsilon_{pq}, \Omega_{pq})}{\partial \epsilon_{ij}}$$

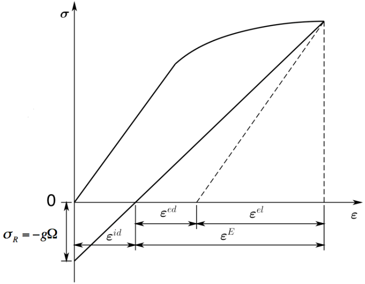
- Conjugation relationships for a damaged isothermal solid [Halm & Dragon, 1998] :

$$\sigma_{ij} = \frac{\partial \Psi_s(\epsilon_{pq}, \Omega_{pq},)}{\partial \epsilon_{ij}} = \frac{\partial \Psi_e(\epsilon_{pq}, \Omega_{pq})}{\partial \epsilon_{ij}} - g\Omega_{ij}$$

$\Rightarrow$ damaged elastic energy + energy required to maintain cracks closed

$$\Psi_s(\epsilon_{pq}, \Omega_{pq}) = \Psi_e(\epsilon_{pq}, \Omega_{pq}) - g\Omega_{ij}\,\epsilon_{ji}$$

$$\Psi_s(\epsilon_{pq}, \Omega_{pq}) = \tfrac{1}{2}\epsilon_{ji}\, D_{e\,ijkl}\,(\Omega_{pq})\;\epsilon_{lk} - g\Omega_{ij}\,\epsilon_{ji}$$



Solid Skeleton Free Energy - Coupled THM Problem [Arson & Gatmiri, 2010, 2012]

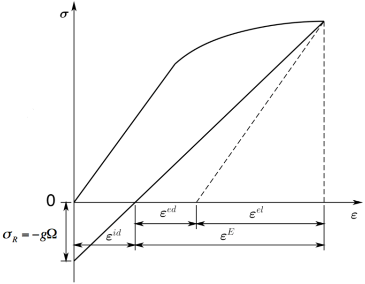
- 3 components of Helmholtz free energy :

$$\Psi_s(\epsilon_{M_{kl}}, \epsilon_{Sv}, \epsilon_{Tv}, \Omega_{kl}) =$$

$$\frac{1}{2}\epsilon_{M_{ji}}\, D_{e_{ijkl}}\left(\Omega_{pq}\right)\, \epsilon_{M_{lk}} + \frac{1}{2}\epsilon_{Sv}\beta_s\left(\Omega_{pq}\right)\epsilon_{Sv} + \frac{1}{2}\epsilon_{Tv}\beta_T\left(\Omega_{pq}\right)\epsilon_{Tv}$$

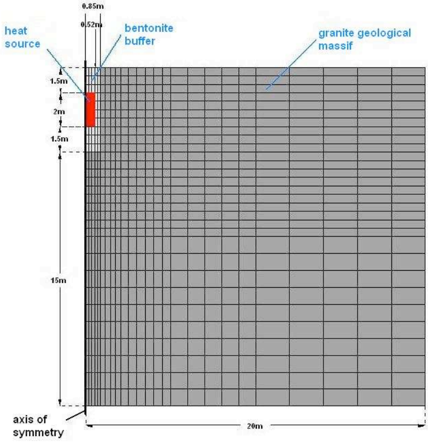
$$-g_M\, \Omega_{ij}\, \epsilon_{M_{ji}} - \frac{g_S}{3}\,\delta_{ij}\,\Omega_{ji}\,\epsilon_{Sv} - \frac{g_T}{3}\,\delta_{ij}\,\Omega_{ji}\,\epsilon_{Tv}$$

- Damaged Rigidities Computed by the Principle of Equivalent Elastic Energy



Influence of the Initial State of Damage [Arson & Gatmiri, 2012]

Kamaishi Experimental Site [Rutqvist et al. 2001]



- depth : 250m
- $T_0 = 12.3^oC$
- granite : $S_{w0} = 1$
- bentonite : $S_{w0} = 0.635$

1. heating source at 100^oC during 8.5 months (259 days)
2. relaxation period of 6 months (183 days)

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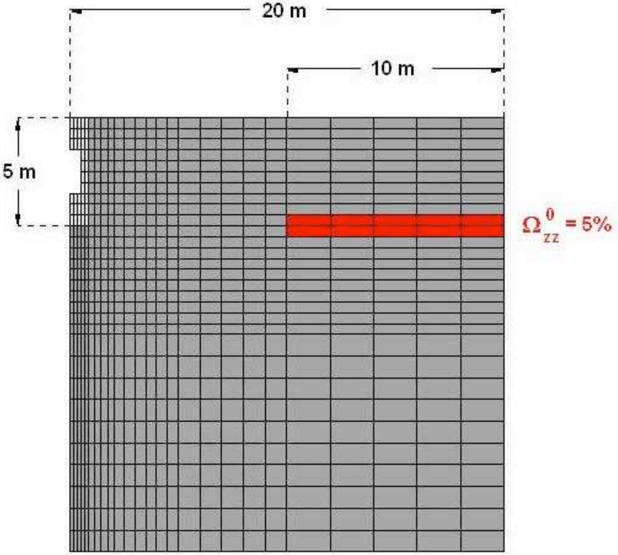
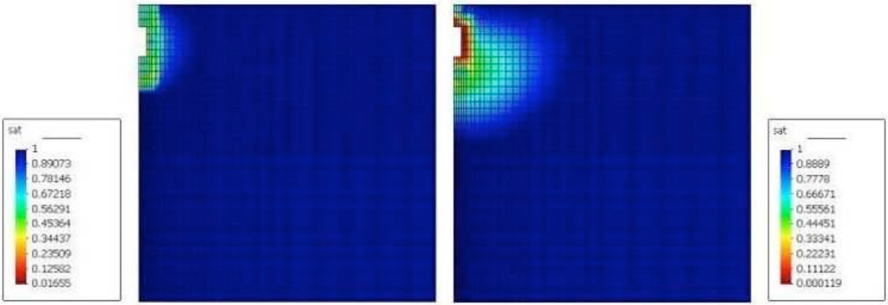


Influence of the Initial State of Damage [Arson & Gatmiri, 2012]

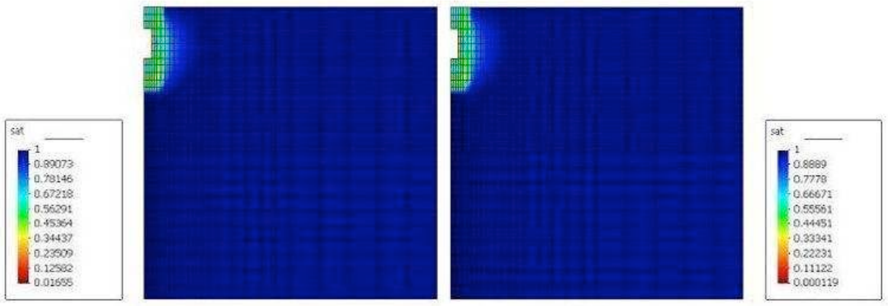
After 10 days of heating



INITIALLY UNDAMAGED ***INITIALLY DAMAGED***

Influence of the Initial State of Damage [Arson & Gatmiri, 2012]

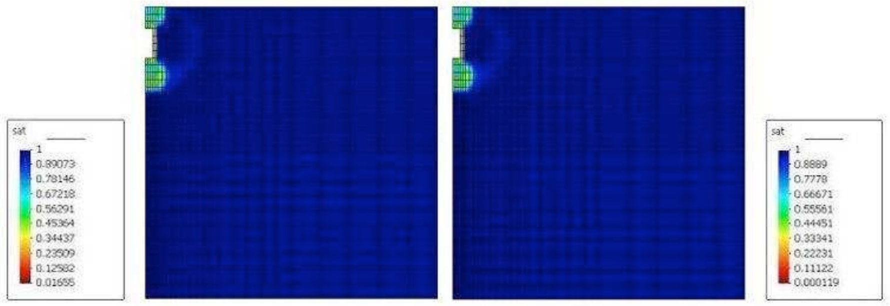
After 20 days of heating



INITIALLY UNDAMAGED ***INITIALLY DAMAGED***

Influence of the Initial State of Damage [Arson & Gatmiri, 2012]

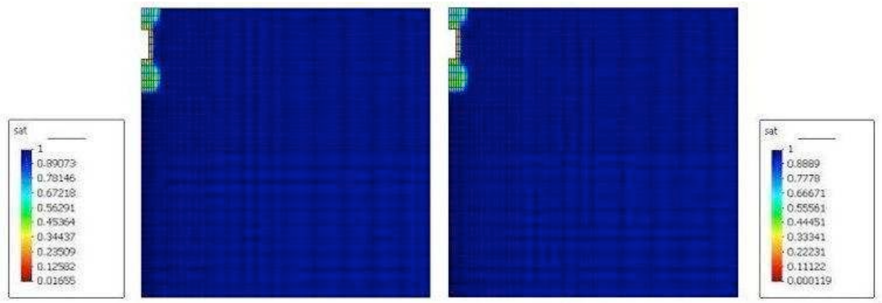
After 30 days of heating



INITIALLY UNDAMAGED ***INITIALLY DAMAGED***

Influence of the Initial State of Damage [Arson & Gatmiri, 2012]

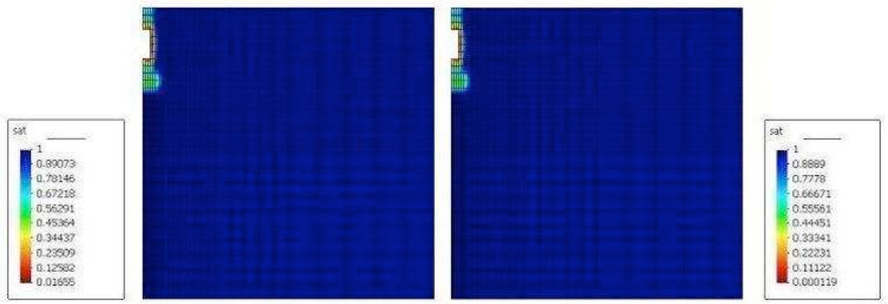
After 50 days of heating



INITIALLY UNDAMAGED ***INITIALLY DAMAGED***

Influence of the Initial State of Damage [Arson & Gatmiri, 2012]

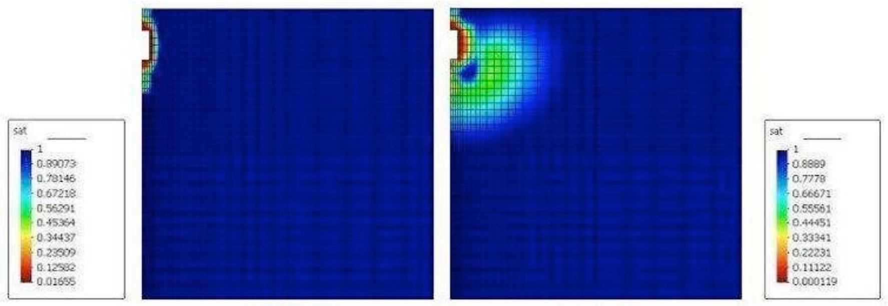
After 259 days of heating



INITIALLY UNDAMAGED ***INITIALLY DAMAGED***

Influence of the Initial State of Damage [Arson & Gatmiri, 2012]

After 259 days of heating and 183 days of relaxation



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Microscopic Processes versus Macroscopic Dilatancy Boundary

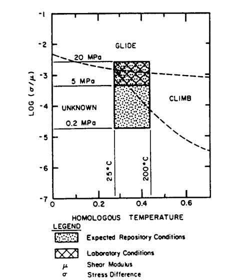
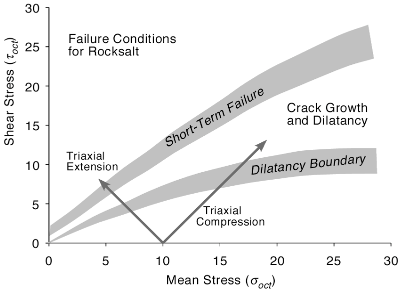


Fig. 17. Deformation mechanism map for WIPP salt [19].

[Senseny et al., 1992]



[Schulze, 2007]

- isochoric dislocation processes
  $\rightarrow$ isochoric viscoplastic deformation
- dilatant micro-cracking
  $\rightarrow$ crack-induced dilatant volumetric deformation
- fluid-assisted Diffusive Mass Transfer (DMT)
  $\rightarrow$ contractant "healing" deformation

Concept of Dilatancy Boundary
[Hunsche & Hampel, 1999 ; Hou, 2003]

- dilatant crack-induced deformation above the boundary
- contractant "healing" deformation below the boundary
- no volume change within the dilatancy boundary zone

"Deformation Healing" versus "Stiffness Recovery"

	Miao et al., 1995	Chan et al., 1998	Hunsche & Hampel, 1999	Hou, 2003	Zhou et al., 2011
Damage Inelastic Deformation	none	stress-induced anisotropic	isotropic dilatancy	stress-induced anisotropic	none (plasticity)
Healing Inelastic Deformation	none	stress-induced anisotropic	isotropic contractancy	stress-induced anisotropic	none
Damaged Elastic Properties	damaged stiffness tensor = internal variable	isotropic softening	none	isotropic softening	isotropic softening
Healed Elastic Properties	isotropic hardening	isotropic hardening	none	isotropic hardening	none

$\Rightarrow$ **Objective :** Model Anisotropic Damage and Temperature-Dependent Healing

1. introduction of internal variables to quantify dissipation induced by damage and healing
2. effect of crack opening and healing on deformation
3. anisotropy induced by cracking (i.e. damage) and healing (i.e. recovery) on the stiffness tensor

Outline of the Damage and Healing Model [Xu, Arson & Chester, 2012]

- Continuum Damage Mechanics : unilateral effects [Chaboche, 1993]
  tensor damage variable $\boldsymbol{\Omega}$ obeying an evolution criterion (f_d) similar to plasticity
  $\Rightarrow$ Kuhn-Tucker consistency equations $\Rightarrow$ damage cannot decrease
  $\Rightarrow$ neutralization of crack-induced damage in compression only (closed cracks)
- Healing Models for Salt Rock [Chan et al., 1998 ; Hunsche & Hampel, 1999 ; Hou, 2003]
  viscoplastic variable controlled by an incremental evolution law ($\dot{\omega}$)
  $\Rightarrow$ can increase (damage growth) and decrease (healing) BUT isotropic

Proposed Modeling Scheme :

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{D_{ed}}\left(\boldsymbol{A}\right) : \dot{\boldsymbol{\epsilon}}, \qquad A_{ij} = \Omega_{ij} - \delta_{ij}\delta_h h$$

"Brittle" crack opening (Kuhn-Tucker conditions apply) + Time-dependent Healing :

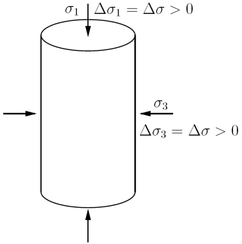
$$f_d\left(\boldsymbol{\Omega}, \boldsymbol{Y_d}\right) = \sqrt{\boldsymbol{Y_d} : \boldsymbol{Y_d}} - C_0 - C_1 \boldsymbol{\Omega} : \boldsymbol{\Omega}, \qquad \dot{h} = \frac{Tr(\boldsymbol{A})pH(p)}{\tau G}$$

DMT healing deformation [Senseny et al., 1992 ; Chan et al., 1998] :

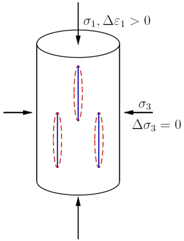
$$\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}^{el} + \dot{\boldsymbol{\epsilon}}^{ed} + \dot{\boldsymbol{\epsilon}}^{id} + \boldsymbol{\delta}\delta_h \dot{\epsilon}^h$$

$$\dot{\epsilon}^h = \frac{\epsilon_v \, pH(p)}{\tau G}, \qquad \dot{\epsilon}^h = C\left(\frac{\sigma_s}{G}\right)^n exp\left(-\frac{Q}{RT}\right)$$

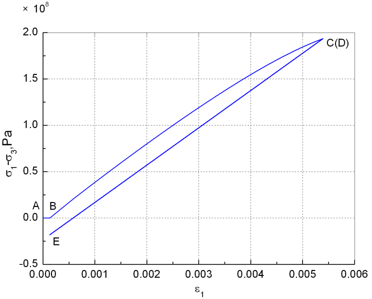
Imposed Stress Path and Resulting Stress/Strain Curves

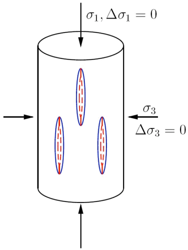


$A \rightarrow B$ **: isotropic confinement**

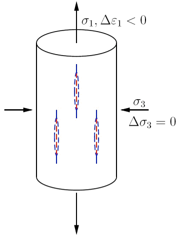


$B \rightarrow C$ **: axial compression**

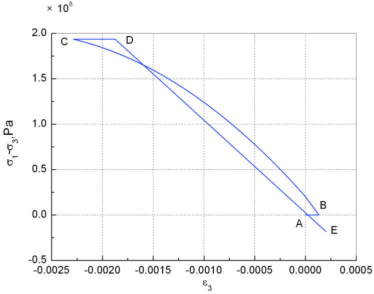




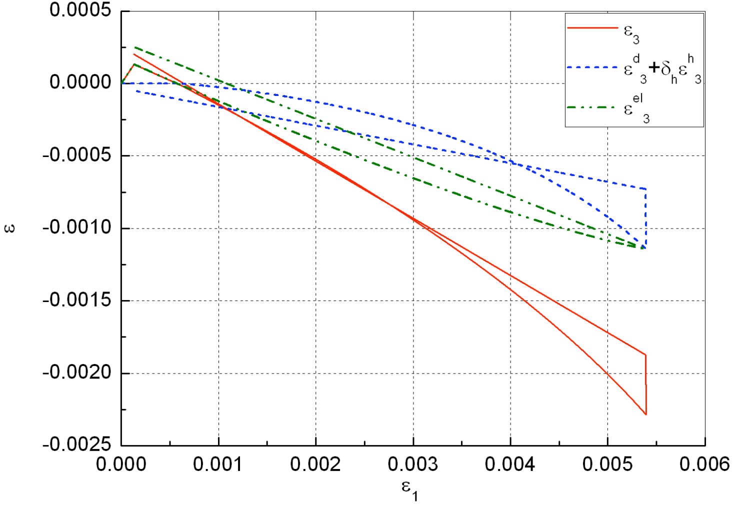
$C \rightarrow D$ **: waiting time (healing)**



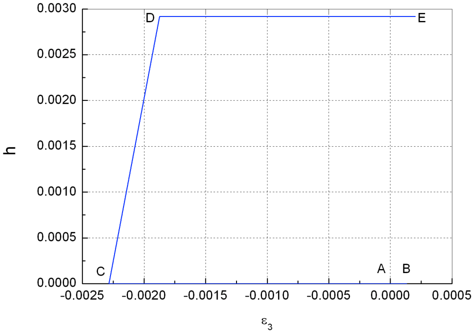
$D \rightarrow E$ **: axial unloading**

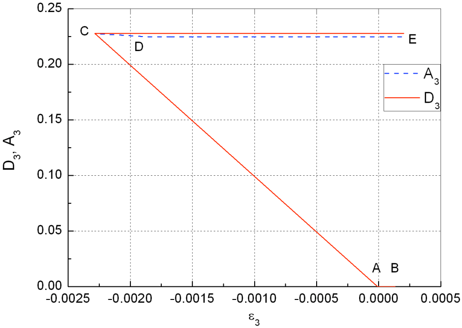


Evolution of the Components of Deformation



Evolution of the Internal Variables





1. **Modeling Damage in Porous Media By A Phenomenological Approach [Arson & Gatmiri, 2008-2011]**
   - Thermodynamic Background
   - A THM Damage Model for Unsaturated Rock : “THHMD Model”
   - Study of Nuclear Waste Disposals with the THHMD Model
2. **Determining Internal Variables to Model Rock Damage and Healing [Xu, Arson & Chester, 2012 ; Arson, Xu & Chester, 2012]**
   - Models of Damage and Healing in Salt Rock : State of the Art
   - A New Damage and Healing Model for Salt Rock
   - Damage and Healing During a Triaxial Compression Test
3. **Relating Damaged Rock Microstructure to Macroscopic Flow and Poro-Elastic Properties [Arson & Pereira ; Pereira & Arson, 2012]**
   - Why is this important to account for the microstructure ?
   - A New Model of Permeability for Cracked Porous Rock
   - Results : Simulation of Triaxial Compression Tests
   - Extension of the Model to Unsaturated Conditions
4. **Homogenized Stiffness Tensor of Damaged Dentin Repaired by Resin Injection [Vennat & Arson, 2012]**

Damage is non local

Damage is non local [Bazant, 1991],
i.e. $\Omega(\boldsymbol{x})$ influences fields of variables at $\boldsymbol{x} + \boldsymbol{dx}$

- **integral formulations** : usually, non-local deformation or non-local energy release rate [Jirasek, 1998]

$$\bar{f}(x) = \int_{V_{tot}} \alpha(x,\xi) f(\xi) d\xi, \qquad \alpha(x,\xi) = \frac{\alpha_\infty(\|x-\xi\|)}{\int_{V_{tot}} \alpha_\infty(\|x-\xi\|) d\xi}$$

- **differential formulations** : usually, introduction of the gradient of deformation [Askes and Sluys, 2002] or the gradient of damage [Frémond & Nedjar, 1996]

$$\epsilon(x+s) = \epsilon(x) + s\,\frac{d\epsilon(x)}{dx} + s^2\,\frac{d^2\epsilon(x)}{dx^2} + o(s^2), \qquad \bar{\epsilon}(x) = \frac{1}{l}\int_{-l/2}^{+l/2} \epsilon(x+s) ds$$

$$\Rightarrow \bar{\epsilon}(x) = \epsilon(x) + \frac{l^2}{24}\frac{d^2\epsilon(x)}{dx^2} + o(l^2)$$

- **microstructure-enriched models** [Mindlin, 1964 ; Germain, 1973(a,b)] : usually second-gradient models (micro-translation), Cosserat models (micro-rotation)

Characteristic length in the expression of the free energy

effective material area introduced in the expression of the free energy $\simeq$ square of the characteristic length [Arson & Gatmiri, 2008]

- energetic term for residual strains after unloading [Halm & Dragon, 1998]

$$\Psi(\epsilon, \Omega) = \frac{1}{2}\epsilon : \boldsymbol{D}(\Omega) : \epsilon - g\boldsymbol{\Omega} : \epsilon$$

- surface energy due to crack opening [Hansen & Schreyer, 1994]

$$\Psi\left(\epsilon, \Omega, \Omega^{\boldsymbol{h}}\right) = \frac{1}{2}\epsilon : \boldsymbol{D}(\Omega)\,\epsilon + H\left(\Omega^{\boldsymbol{h}}\right) + \gamma_D\,\boldsymbol{\Omega} : \boldsymbol{\Omega}$$

- energy related to damage influence zone [Frémond & Nedjar, 1998]

$$\Psi(\epsilon, \omega, \nabla\omega) = \frac{1}{2}\epsilon : \boldsymbol{D_0} : \epsilon + W(1-\omega) - M\left[log(|\omega|) - \omega + 1\right] + \frac{k}{2}(\nabla\omega)^2$$

Damaged Permeability Model [Arson & Pereira, 2012] : Assumptions

- Cracks do not interact $\Rightarrow$ damage = crack-density tensor [Kachanov, 1992]

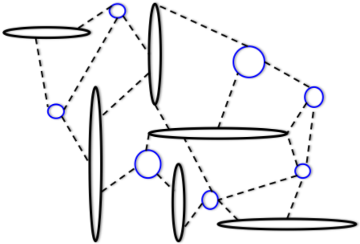
$$\boldsymbol{\Omega} = \sum_{k=1}^{3} d^k \boldsymbol{n^k} \otimes \boldsymbol{n^k}$$

- Cracks do not intersect but are connected to the natural pores :

$$\boldsymbol{K_w} = \boldsymbol{K_w^0} + \boldsymbol{K_w^c}$$

- Cracks and natural pores are connected, but do not overlap :

$$V_v = V_p + V_c$$

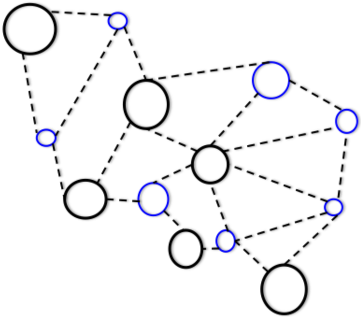


Damaged Permeability Model [Arson & Pereira, 2012] : Assumptions

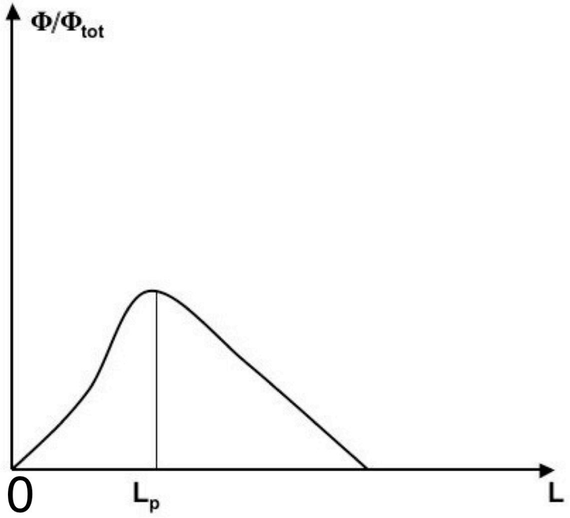
- Flow in the natural pores and cracks is modeled as a laminar flow in parallel cylinders.

  For an isotropic model [Garcia-Bengochea et al., 1979] :

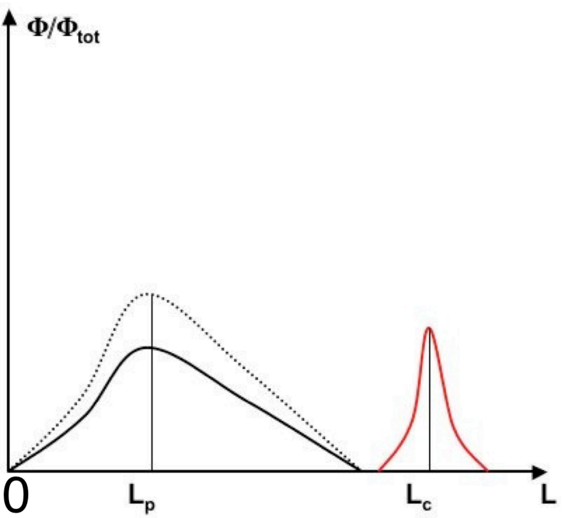
$$k_w = \frac{\gamma}{8\mu}\Phi\frac{1}{\int_0^\infty f(r)dr}\int_0^\infty f(r)r^2dr$$



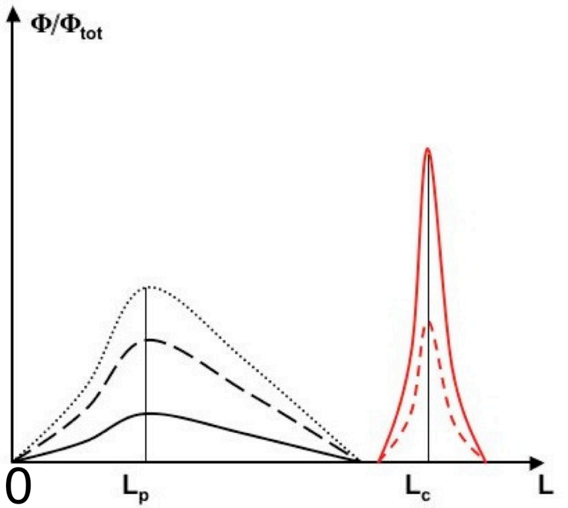
Expected Evolution of the Pore Size Distribution Curve with Cracking



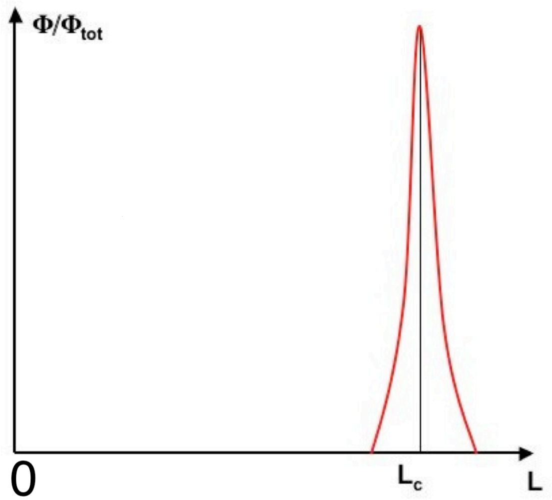
Expected Evolution of the Pore Size Distribution Curve with Cracking



Expected Evolution of the Pore Size Distribution Curve with Cracking

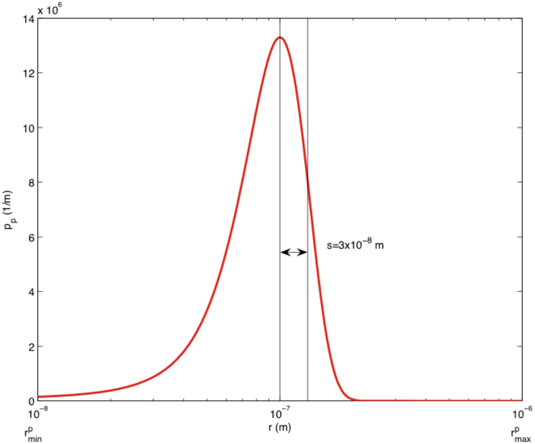


Expected Evolution of the Pore Size Distribution Curve with Cracking



Size Distribution of Natural Pores

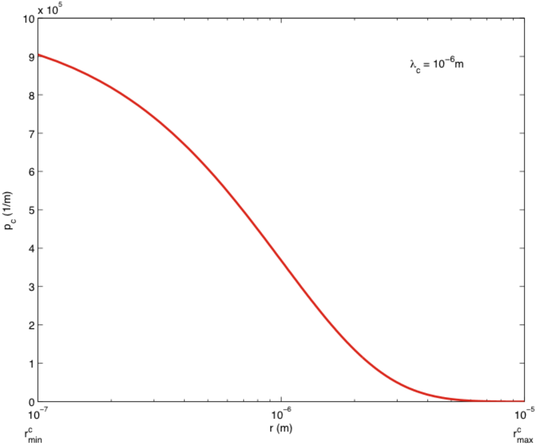
Natural Pores : bell-shaped distribution [Van Genuchten, 1980 ; Alves et al., 1996]



$$p_p(r) = \begin{cases} \frac{1}{s\sqrt{2\pi}} exp\left(-\frac{(r-m)^2}{2\,s^2}\right) & if \quad r^p_{min} \leq r \leq r^p_{max} \\ 0 \quad if \quad r < r^p_{min} \quad or \quad if \quad r > r^p_{max} \end{cases}$$

Size Distribution of Cracks

Cracks : exponential distribution [Maleki, 2004]



$$p_c(r) = \begin{cases} \frac{1}{\lambda_c} exp\left(-\frac{r}{\lambda_c}\right) & if \quad r^c_{min} \leq r \leq r^c_{max} \\ 0 \quad if \quad r < r^c_{min} \quad or \quad if \quad r > r^c_{max} \end{cases}$$

PSD Parameters & Porous Volumes [Arson & Pereira, 2012]

Volumetric frequency of the pores of radius r :

$$f(r) = L\,\alpha(r)\,\pi\,r^2 \qquad \alpha(r) = \alpha_p(r) + \alpha_c(r)$$

Micro/macro relationship for a unit REV ($L = 1$) :

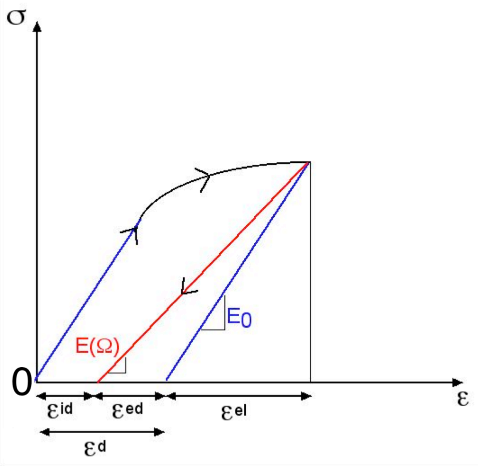
$$\int_{r^p_{min}}^{r^p_{max}} \alpha_p(r)\pi r^2 dr = V_p \qquad \int_{r^c_{min}}^{r^c_{max}} \alpha_c(r)\pi r^2 dr = V_c$$

r^p_{min}, r^p_{max}, r^c_{min}, r^c_{max} : min. and max. radius values for natural pores and cracks
α_p : frequency of occurrence of natural pores of radius r in the REV
α_c : frequency of occurrence of cracks of radius r in the REV

$$\pi N_p \int_{r^p_{min}}^{r^p_{max}} p_p(r) r^2 dr = V_p \qquad \pi N_c \int_{r^c_{min}}^{r^c_{max}} p_c(r) r^2 dr = V_c$$

N_p, N_c : number of natural pores and cracks in the REV

Porous Volumes & Thermodynamic Variables [Arson & Pereira, 2012]



$$\Delta V_p = -Tr\left(\epsilon^{\boldsymbol{el}}\right) \qquad \Delta V_c = -Tr\left(\epsilon^{\boldsymbol{d}}\right) \qquad \boldsymbol{D}(\Omega) : \epsilon^{\boldsymbol{id}} = -g\Omega$$

Resolution Algorithm : from Macro to Micro Variables

1. update the mean radius of natural pores $m^{(k)}$:

$$V_p{}^{(k)} = \pi\, N_p \int_{r^p_{min}}^{r^p_{max}} \left(\frac{1}{s\sqrt{2\pi}} exp\left(-\frac{(r-m^{(k)})^2}{2\, s^2} \right) \right) r^2 dr$$

N_p and s are fixed parameters, determined in the algorithm initialization

2. update the number of cracks present in the REV $N_c^{(k)}$

$$V_c{}^{(k)} = \pi\, N_c^{(k)} \int_{r^c_{min}}^{r^c_{max}} \frac{1}{\lambda_c} exp\left(-\frac{r}{\lambda_c} \right) r^2 dr$$

λ_c is a fixed parameter, determined in the algorithm initialization

3. update $\alpha_p^{(k)}(r)$ and $\alpha_c^{(k)}(r)$, update the "volumetric frequency" $f^{(k)}(r)$:

$$f^{(k)}(r) = \left(\alpha_p^{(k)}(r) + \alpha_c^{(k)}(r) \right) \pi r^2$$

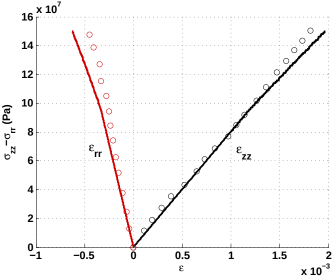
4. update hydraulic conductivity $k_w{}^{(k)}$:

$$k_w{}^{(k)} = \frac{\gamma}{8\mu} \left(\Phi_0 + Tr\left(-\epsilon^{(k)} \right) \right) \frac{1}{\int_0^\infty f^{(k)}(r) dr} \int_0^\infty f^{(k)}(r) r^2 dr$$

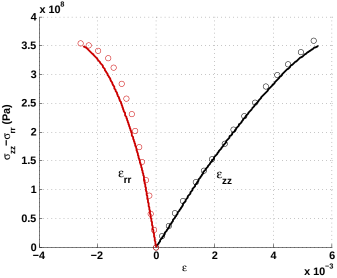
Simulation of Triaxial Compression Tests (Granite, $\sigma_c = 0$) [Arson & Pereira, 2012]

E (Pa)	ν (-)	g (Pa)	C_0 (Pa)	C_1 (Pa)	e_0 (-)
$8.01\,10^{10}$	0.28	$-3.3\,10^{8}$	$1.1\,10^{5}$	$2.2\,10^{6}$	0.008
r^p_{min} (μm)	r^p_{max} (μm)	r^c_{min} (μm)	r^c_{max} (μm)		
0.01	1	0.1	10		

Reference Results [Halm and Dragon, 2002 ; Arson and Gatmiri, 2010]

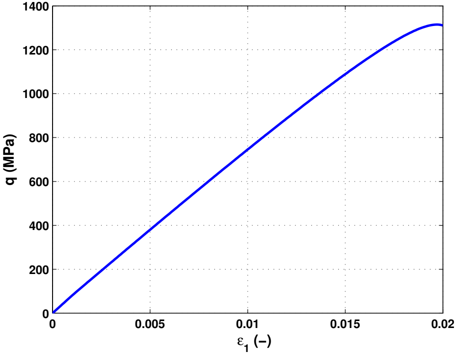


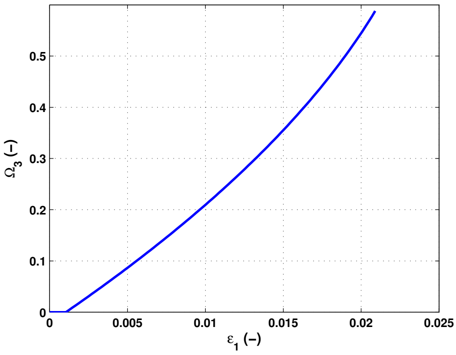
$\sigma_c \;=\; \mathbf{0\,MPa}$



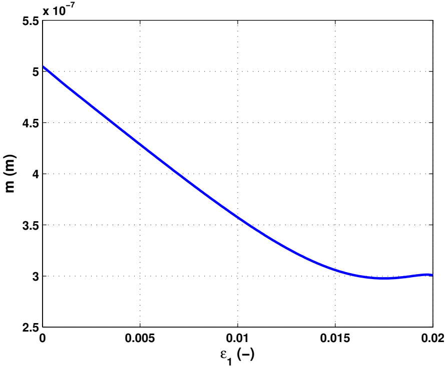
$\sigma_c \;=\; \mathbf{20\,MPa}$

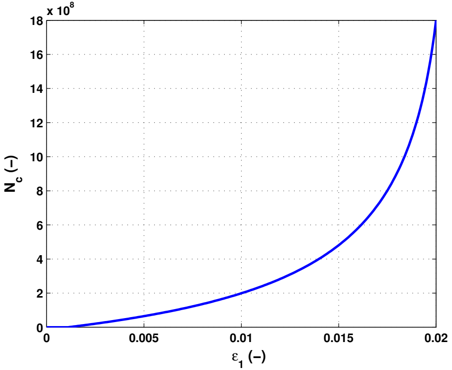
Deviatoric Stress and Damage Evolutions



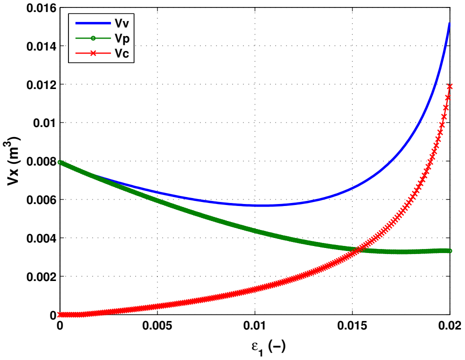


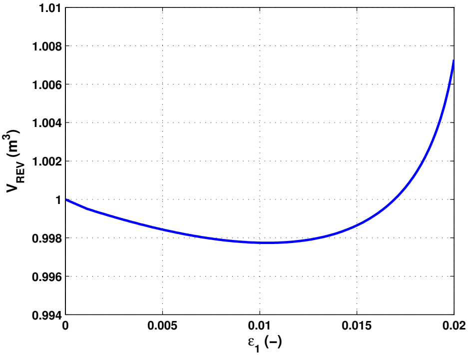
Variations of the Model Variables m and N_c



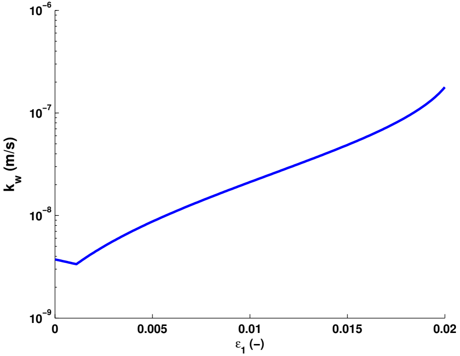


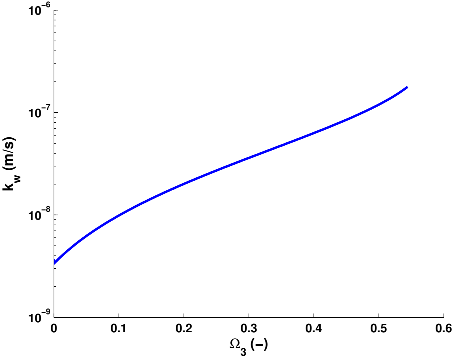
Porous Volume Changes





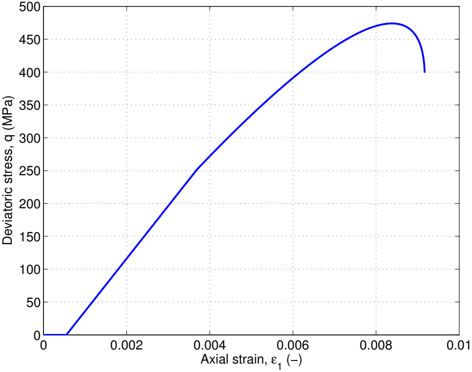
Impact on Permeability

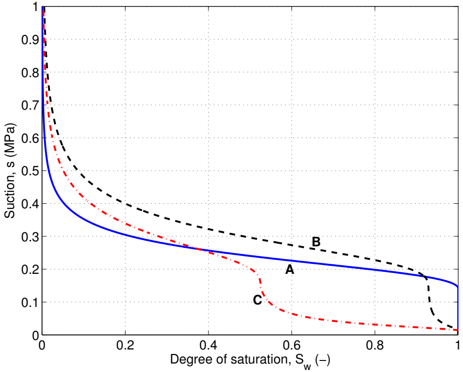




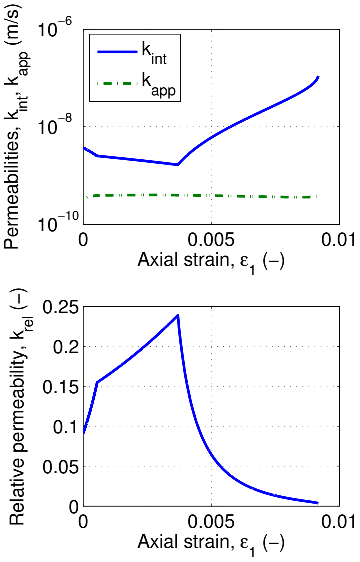
Impact on Degree of Saturation

Triaxial Compression Test with control of capillary pressure
$(p_c = 300\, kPa)$

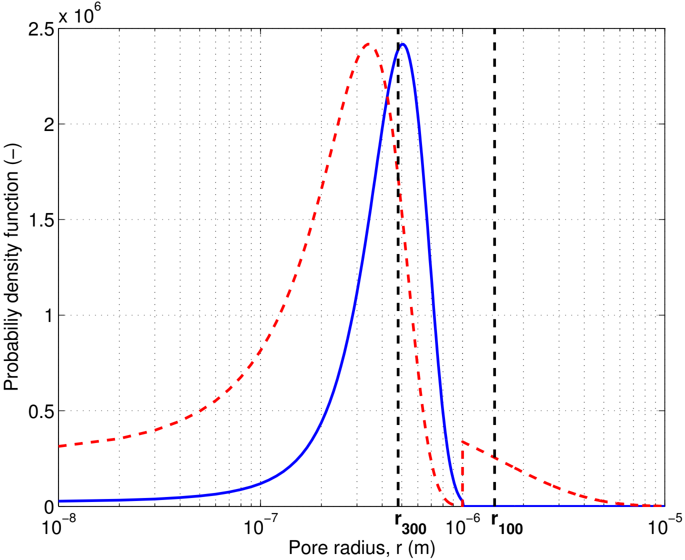




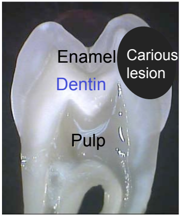
Impact on Intrinsic and Relative Permeabilities

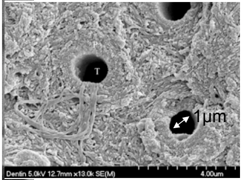


Impact on Pore Size Distributions



Tooth Structure

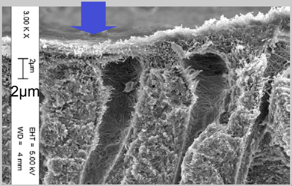




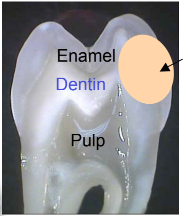
Intact dentin

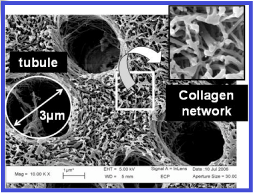
- Carious lesion removal

- Superficial demineralization : hydroxyapatite removal on a few microns



Dentin superficially demineralized





Demineralized dentin

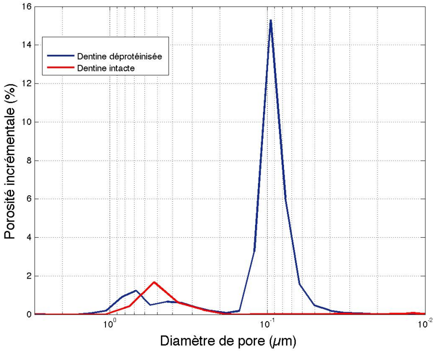
Picture : courtesy of Dr. E. Vennat, Ecole Centrale Paris, France

Research Problems

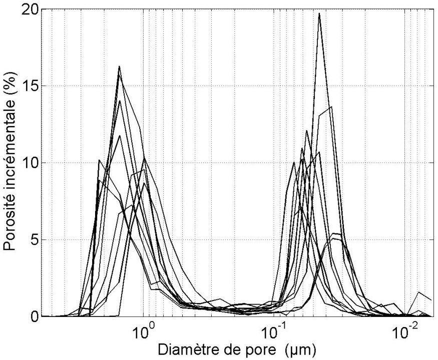
- compression strength of intact, demineralized and injected dentin ?
- dentin stiffness tensor ?
  1. stiffness homogenization of collagen fibers : protein matrix + hydroxyapatite crystals / water / resin ? ?
  2. stiffness homogenization of Inter-Tubular Dentin (ITD) : fibers + hydroxyapatite crystals / water ? ? ? resin ? ? ?
  3. stiffness homogenization of ITD + Peri-Tubular Dentin (PTD, considered homogeneous) + Tubules (lumen / resin)
- **micro-structure of intact, demineralized and injected dentin ?** periodic composite ? mixture of solid and fluid constituents ?
- homogenization scheme ?
- adhesion remaining dentin / resin ?

First Micro-Porosimetry Observations

Intact dentin & Dentin with No Collagen



Demineralized Dentin



Pictures : courtesy of Dr. E. Vennat, Ecole Centrale Paris, France

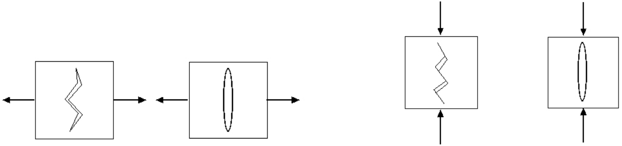
Current Research Topics

Constitutive Modeling of Porous Media Subject to Damage and Thermo-Hydro-Chemo-Mechanical Couplings

1. Damage and Healing in Rock : Fault Mechanics, Hydraulic Fracturing, Geostorage in Salt Rock, Waste Repositories in Clay Rock
2. Poromechanics Applied to Energy Engineering : Heat Exchanger Piles, Thermo-Osmotic Flow around Nuclear Waste Disposals, CO_2 sequestration
3. Combination of Continuum and Particulate Mechanics for Sustainable Infrastructure : Ballast Particle Crushing, Scour & Erosion around Bridge Piers
4. Damage and Fatigue in Materials Other than Rock : Elastomeric Bearings, Cement Paste, Teeth...

Thanks !
Any question ?

Damage Evolution Law : Extension of the Concept of Tensile Strain [Arson & Gatmiri, 2009]



- influence of tensile mechanical stress, thermal expansion and capillary pore shrinkage :

$$Y^{+}_{d1\,ij} = g_M\, \epsilon^{+}_{M\,ij} + \frac{g_S}{3}\epsilon^{-}_{Sv}\delta_{ij} + \frac{g_T}{3}\epsilon^{+}_{Tv}\delta_{ij}$$

- a unique damage criterion :

$$f_d\left(Y_{d\,pq}, \Omega_{pq}\right) = \sqrt{\frac{1}{2}\,Tr\left(Y^{+}_{d_1\,ij}\, Y^{+}_{d_1\,ji}\right)} - C_0 - C_1\delta_{ij}\,\Omega_{ji}$$

- after applying the consistency rules :

$$d\Omega_{ij} = d\lambda_d \frac{\partial f_d\left(Y_{d\,pq}, \Omega_{pq}\right)}{\partial Y^{+}_{d_1\,ji}}$$

Impact of Damage on Intrinsic Permeability [Arson & Gatmiri, 2010, 2012]

Liquid Water Flow :

$$V_{w_i} = -\frac{\Psi_R(\theta_w)}{\sigma(T_{ref})} \frac{d\sigma(T)}{dT} K_{w_{ij}} \nabla(T)_j + \frac{1}{\gamma_w} \frac{\sigma(T)}{\sigma(T_{ref})} K_{w_{ij}} \nabla(s)_j - K_{w_{ij}} \nabla(z)_j$$

Influence of Damage on **intrinsic permeability** :

$$K_{w_{ij}} = k_r(S_w,\ T)\, K_{int_{ij}}(n, \Omega_{pq})$$

$$K_{w_{ij}} = k_r(S_w,\ T) \left[K_{ij}^{intact}(n^{rev}) + K_{ij}^{dg}(n^{frac}, \Omega_{pq}) \right]$$

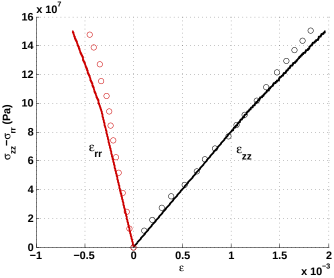
Assumption : laminar flow in the 3 equivalent cracks of the REV [Shao et al. 2005] :

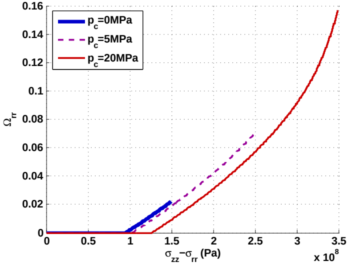
$$K_{ij}^{dg}\left(n^{frac}, \Omega_{rs}\right) = \frac{\pi^{-2/3}\gamma_w}{12\,\mu_w(T_{ref})} \chi^{4/3} b^2 \sum_{k=1}^{3} \left(d^k\right)^{5/3} \left(\delta_{ij} - n_i^k n_j^k\right)$$

b : internal length parameter

Triaxial Compression Tests [Arson & Gatmiri, 2010]

Mechanical Tests - “dry” granite





$\mathbf{p_c = 0\,MPa}$

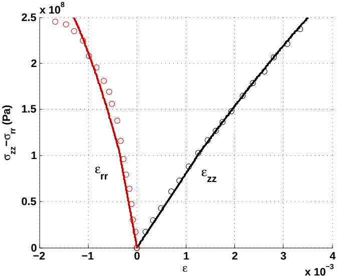
$\Omega_{rr} = \Omega_{\theta\theta}\,, \quad \Omega_{zz} = 0$

Granite Main Material Parameters [Halm and Dragon 2002]

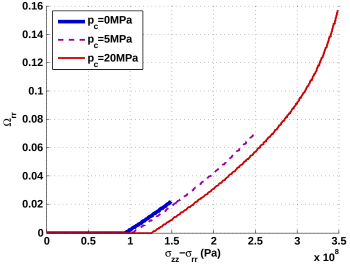
E_0 (Pa)	ν_0	β_S^0 (Pa)	β_T^0 (Pa)	
$8.01\,10^{10}$	0.28	$6.07\,10^{11}$	$6.07\,10^{11}$	
C_0 (Pa)	C_1 (Pa)	g_M (Pa)	g_S (Pa)	g_T (Pa)
$1.1\,10^5$	$2.2\,10^6$	$-3.3\,10^8$	0	0

Triaxial Compression Tests [Arson & Gatmiri, 2010]

Mechanical Tests - "dry" granite



$\mathbf{p_c} = \mathbf{5\,MPa}$



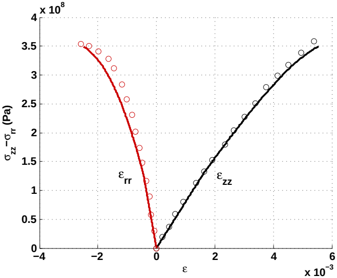
$\Omega_{rr} = \Omega_{\theta\theta}\,, \quad \Omega_{zz} = 0$

Granite Main Material Parameters [Halm and Dragon 2002]

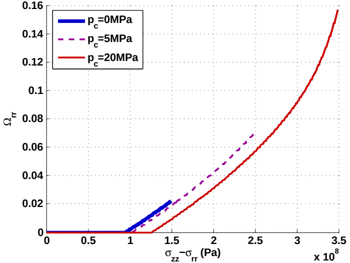
E_0 (Pa)	ν_0	β_S^0 (Pa)	β_T^0 (Pa)	
$8.01\ 10^{10}$	0.28	$6.07\ 10^{11}$	$6.07\ 10^{11}$	
C_0 (Pa)	C_1 (Pa)	g_M (Pa)	g_S (Pa)	g_T (Pa)
$1.1\ 10^5$	$2.2\ 10^6$	$-3.3\ 10^8$	0	0

Triaxial Compression Tests [Arson & Gatmiri, 2010]

Mechanical Tests - “dry” granite



$\mathbf{p_c} = \mathbf{20\,MPa}$



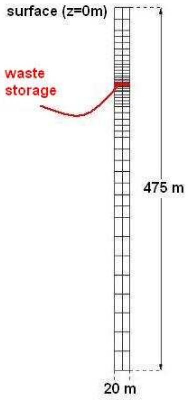
$\Omega_{rr} = \Omega_{\theta\theta}, \quad \Omega_{zz} = 0$

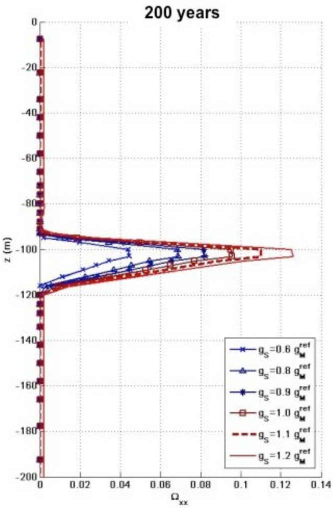
Granite Main Material Parameters [Halm and Dragon 2002]

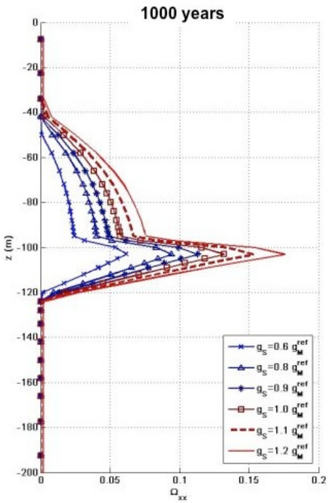
E_0 (Pa)	ν_0	β_S^0 (Pa)	β_T^0 (Pa)	
$8.01\ 10^{10}$	0.28	$6.07\ 10^{11}$	$6.07\ 10^{11}$	
C_0 (Pa)	C_1 (Pa)	g_M (Pa)	g_S (Pa)	g_T (Pa)
$1.1\ 10^5$	$2.2\ 10^6$	$-3.3\ 10^8$	0	0

Influence of the Damage Parameters [Arson & Gatmiri, 2010]

[Pollock, 1986]







$S_{w0} = 0.15$

Influence of the Internal Length Parameter [Arson & Gatmiri, 2010]

$$K_{2_{ij}}^{dg}\left(n^{frac},\Omega_{rs}\right) = \frac{\pi^{-2/3}\gamma_w}{12\,\mu_w(T_{ref})}\,\chi^{4/3}\,b^2 \times \sum_{k=1}^{3}\left(d^k\right)^{5/3}\left(\delta_{ij} - n_i^k\,n_j^k\right)$$

$K_{ij}^{dg} = K_{w,\,dg}^{max}\,\delta_{ij}$ for $\Omega_{ij} = 0.95\delta_{ij}$

$\Rightarrow$ computation of b